

CAPM - Challenge Problems

Thursday, September 28, 2017 10:06 AM

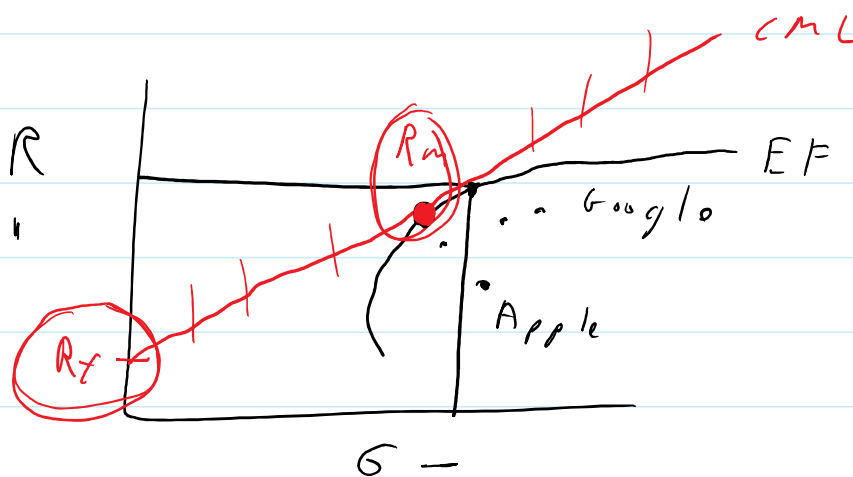
$$R_i = R_f + \beta_i \times (R_m - R_f)$$

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

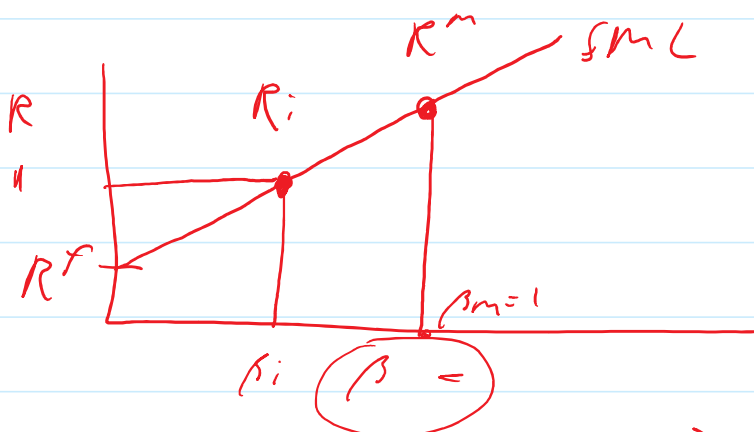
$$\sigma_{im} = \rho_{im} \sigma_i \sigma_m$$

$$RP_i = R_i - R_f$$

$$\text{Sharpe} := \frac{R_i - R_f}{\sigma_i}$$



R_i vs R_m



β_i ($\beta =$) \rightarrow

Prob 1

$$R_M = 7.5\%$$

$$R_f = 1\%$$

$$\rho = 0.1$$

$$\sigma_A^2 = 0.01$$

$$\sigma_M = 9\%$$

$$\begin{aligned} ? = R_A &= R_f + \beta \times (R_M - R_f) \\ &= 1\% + \beta \times (7.5\% - 1\%) = 1.7\% \end{aligned}$$

$$\beta = \frac{\sigma_{AM}}{\sigma_M^2} = \frac{\rho \cdot \sigma_A \cdot \sigma_M}{\sigma_M^2}$$

$$\sigma_{AM} = \rho \cdot \sigma_A \cdot \sigma_M$$

$$= \boxed{\rho} \cdot \frac{\sigma_A}{\sigma_M} = 0.1 \frac{\sqrt{0.01}}{.09} = .11$$

PS

$$0.35 = \frac{R_A - R_f}{\sigma_A}$$

$$0.5 = \frac{R_M - R_f}{\sigma_M}$$

$$R_A = R_f + \beta \cdot (R_m - R_f)$$

$$R_A - R_f = \beta \cdot (R_m - R_f)$$

$$\frac{R_A - R_f}{\sigma_A} = \rho \frac{\sigma_A}{\sigma_m} \frac{(R_m - R_f)}{\sigma_m}$$

$$\frac{R_A - R_f}{\sigma_A} = \rho \frac{R_m - R_f}{\sigma_m}$$

$$0.35 = \rho \cdot 0.5 \Rightarrow \rho = \frac{.35}{.5} = .7$$

$$\beta = \frac{\sigma_{AM}}{\sigma_m^2} = \frac{\rho \cdot \sigma_A \cdot \sigma_m}{\sigma_m^2}$$

$$\sigma_{AM} = \rho \cdot \sigma_A \cdot \sigma_m$$

PG

$$R_A = 10\% \quad \beta = 1.4$$

$$R_{Pm} = 6\%$$

$$R_A = R_f + \beta \cdot (R_m - R_f)$$

$$10\% = R_f + 1.4 \times 6\%$$

$$R_f = 10\% - 1.4 \times 6\% = 1.6\%$$

p7

$$\rho = .8 \quad \beta = .67$$

$$RP_M = 6.5\% = R_M - R_f \quad \text{Sharpe} = 0.43 = \frac{R_M - R_f}{\sigma_M}$$

$$0.43 \cdot \sigma_M = R_M - R_f = 6.5\%$$

$$\sigma_M = \frac{6.5\%}{0.43} = 15\%$$

$$\sigma_A^2 = ? = 0.0160$$

$$\sigma_{AM} = \rho \cdot \sigma_A \sigma_M$$

$$\beta = \frac{\sigma_{AM}}{\sigma_M^2} = \rho \cdot \frac{\sigma_A \sigma_M}{\sigma_M^2} = \rho \cdot \frac{\sigma_A}{\sigma_M}$$

$$0.67 = .8 \cdot \frac{\sigma_A}{15\%}$$

$$\sigma_A = \frac{.67 \cdot .15}{.8} = 13\%$$